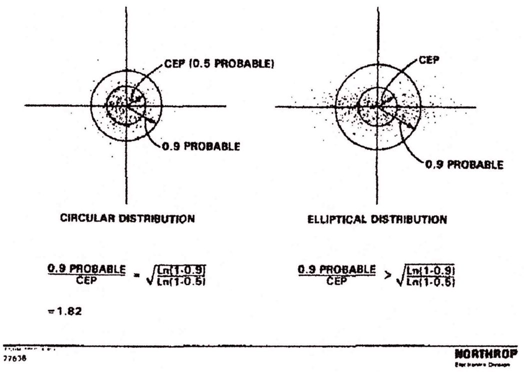
For the case illustrated by the scatter diagrams of Figure 2, there is negative correlation between the down range, X and altitude, Z, errors. The Pxz (=Pzx of the covariance matrix would be negative. This negative correlation between downrange and altitude error is quite typical of guidance induced errors. Because of it, an optimal integrated fuze implementation can use a radar altitude fix to gain insight into downrange error, change the time at which the burst would occur, and thereby improve the probability of target damage. The covariance matrix representation of the distribution has a very convenient computative advantage. You can add covariance matrices. Given covariance matrices for various (uncorrelated error distributions, the covariance matrix for the combined distriubtion is simply their sum. This can't be done with distribution functions.

Figure 1

EVEN IN 2-DIMENSIONS – THE CEP CONVENTION IS FLAWED



CIRCULAR DISTRIBUTION

ELLIPTICAL DISTRIBUTION

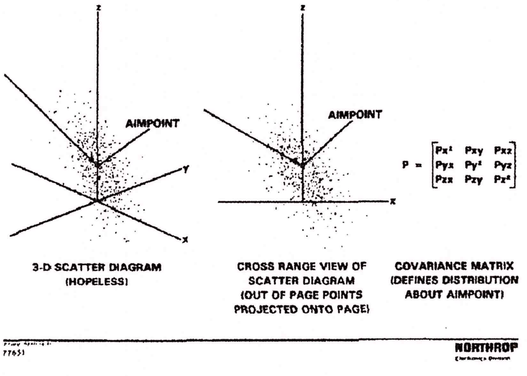
$$\frac{0.9 \text{ PROBABLE}}{\text{CEP}} = \sqrt{\frac{\text{Ln}(1-0.9)}{\text{Ln}(1-0.5)}}$$

$$= 1.82$$

$$\frac{0.9 \text{ PROBABLE}}{\text{CEP}} > \sqrt{\frac{\text{Ln}(1-0.9)}{\text{Ln}(1-0.5)}}$$

Figure 2

REPRESENTATIONS OF 3-DIMENSIONAL PROBABILITY DISTRIBUTION



$$P = \begin{bmatrix} Px^2 & Pxy & Pxz \\ Pyx & Py^2 & Pyz \\ Pzx & Pzy & Pz^2 \end{bmatrix}$$

3-D SCATTER DIAGRAM (HOPELESS)

CROSS RANGE VIEW OF SCATTER DIAGRAM (OUT OF PAGE POINTS PROJECTED ONTO PAGE)

COVARIANCE MATRIX (DEFINES DISTRIBUTION ABOUT AIMPOINT)

Classically, probabilities are defined in terms of integrals of distribution functions, not covariance matrices. However, the probabilities can also be formed using a Monte Carlo analysis wherein the three (pseudo) random variables used have correlated distributions computed from a covariance matrix. The Monte Carlo approach is simple, it works, and it may in fact, be the only practical way to solve the problem.

The covariance matrix may be used to represent an "a priori" distribution, that due to weapon system error sources that develop prior to the time the re-entry vehicle reaches the target area.

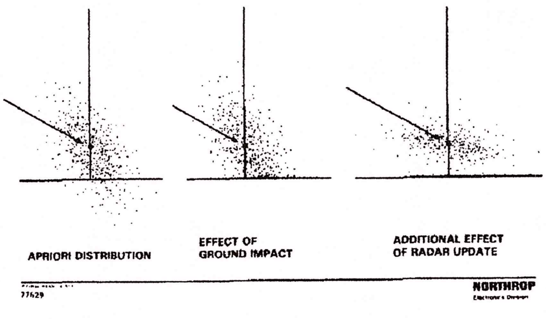
If there were no interraction with the ground (to be discussed shortly) this a priori distribution could represent the actual three-dimensional burst point position error. The burst would be controlled by a timer in the fuze that is set by the guidance software for the time at which the re-entry vehicle should normally reach the aimpoint. Additionally, the a priori distribution may include the effect of a "path length fuze". If used, the path length fuze would employ a calibrated accelerometer to correct the timer set point for the effects of off nominal values of atmospheric density and re-entry vehicle drag coefficient. The a priori distribution may then be changed in two ways; as illustrated in Figure 3. First, in reality, the burst can't occur below the ground (at least significantly below it). Although the aimpoint is above the surface, the a priori distribution may be such that there is a significant probability that the burst point occurs below the surface. The Monte Carlo method of analysis makes it easy to modify the distribution to account for ground impact. It is done by correcting the burst position for each below-the-ground sample. The corrected position is computed by backing out from the underground burst point to the surface along the direction of the re-entry flight path. This is easy to do with a Monte Carlo simulation.

We have assumed that the burst does occur at the earth's surface in these cases as a result of a contact fuze. Alternatively, it could be assumed that those that impact do not detonate.

The second modification to the a priori distribution comes about as a result of a radar update, if used. Some time prior to reaching the a priori burst point, a single radar altitude measurement may be made.

Figure 3

THE REAL BURST DISTRIBUTION IS AFFECTED BY THE LOCATION OF THE GROUND AND RADAR FUZE UPDATE. IT CAN NO LONGER BE REPRESENTED BY A COVARIANCE MATRIX.



APRIORI DISTRIBUTION

EFFECT OF GROUND IMPACT

ADDITIONAL EFFECT OF RADAR UPDATE